

On the Infrared Behaviour of Landau Gauge Yang-Mills Theory with Differently Charged Scalar Fields

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Abstract. Recently it has been argued that infrared singularities of the quark-gluon vertex of Landau gauge QCD can confine static quarks via a linear potential. It is demonstrated that the same mechanism also may confine fundamental scalar fields. This opens the possibility that within functional approaches static confinement is an universal property of the gauge sector even though it is formally represented in the functional equations of the matter sector. The colour structure of Dyson-Schwinger equations for fundamental and adjoint scalar fields is determined for the gauge groups $SU(N)$ and $G(2)$ exhibiting interesting cancelations purely due to colour algebra.

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MOTIVATION

QCD Green's functions are unique in the sense that they may provide a look onto the detailed relation between hadron phenomenology and fundamental properties of QCD. On the one hand, they can be used to describe hadrons in terms of glue and quarks. To this end one employs either the Bethe-Salpeter equations for mesons [1] or the Poincaré covariant Faddeev equation for baryons [2]. On the other hand, they provide insight into the non-perturbative structure of QCD, and hereby most noticeably into Dynamical Breaking of Chiral Symmetry ($D\chi SB$), the $U_A(1)$ anomaly, and confinement. Hereby it is interesting to note that $D\chi SB$ does not only imply the dynamical generation of quark masses but also of Lorentz-scalar couplings between quarks and gluons, see [3, 4] and references therein. The $U_A(1)$ anomaly can be related to the infrared behaviour of Green's function [5, 6] and thus to confinement.

As for the infrared behaviour of Landau gauge QCD Green's functions it has become evident that both, Functional Renormalization Group and Dyson-Schwinger equations, display two different types of solutions. There is exactly one unique solution with powerlaw infrared behaviour which is called scaling solution [7]. On the other hand, there is an one-parameter family of solutions with an infrared constant gluon propagator and infrared-trivial vertex functions, for a discussion see [8] and references therein. A similar situation is known since quite some time in Coulomb gauge where it occurs when one applies variational methods [9]. In Landau gauge, however, this has been noticed only recently, see *e.g.* [8, 10, 11].

A possible resolution of this apparent ambiguity might be that even the minimal Landau gauge needs some additional input to be fixed also non-perturbatively [12]. In the following we will assume that at least one gauge exists which also allows for the scaling solution.

LANDAU GAUGE YANG-MILLS GREEN'S FUNCTIONS

The derivation and main characteristics of the scaling solution is summarized in a contribution to the Proceedings of the Confinement Conference 2008 [13] to which the reader is referred for more details. The following general infrared behaviour for one-particle irreducible Green functions with $2n$ external ghost legs and m external gluon legs is in the scaling solution given as [14, 15]:

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n-m)\kappa + (1-n)(d/2-2)} \quad (1)$$

where d is the space-time dimension and κ is one yet undetermined parameter. It fulfills some very general inequalities [16, 17] which can be summarized as $0.5 \leq \kappa < 23/38$. With some assumptions on the ghost-gluon vertex its value can be determined to be $\kappa = 0.595$ [18]. A further important property is that there are additional divergences when only some of the momenta of the n -point functions are vanishing [19].

Eq. (1) especially entails that the ghost propagator and the three- and four-point gluon vertex functions are infrared divergent. As we will see below this has then profound consequences for the quark-gluon vertex as well as for vertices involving fundamentally charged matter.

QUARK-GLUON VERTEX

Due to the infrared suppression of the gluon propagator, present in the scaling and in the decoupling solutions, quark confinement cannot be generated by any type of gluon exchange together with an infrared-bounded quark-gluon vertex. To proceed it turns out to be necessary to study the Dyson-Schwinger equation for the quark propagator together with the one for the quark-gluon vertex in a self-consistent way [3]. Hereby a drastic difference of the quarks as compared to Yang-Mills fields has to be taken into account: As they possess a mass, and as $D\chi SB$ does occur, the quark propagator will always approach a constant in the infrared.

The fully dressed quark-gluon vertex can be expanded in twelve linearly independent Dirac tensors. Half of the coefficient functions would vanish if chiral symmetry were realized in the Wigner-Weyl mode. From a solution of the Dyson-Schwinger equations we infer that these “scalar” structures are, in the chiral limit, generated non-perturbatively together with the dynamical quark mass function in a self-consistent fashion. Thus dynamical chiral symmetry breaking manifests itself not only in the propagator but also in the quark-gluon vertex.

From an infrared analysis one obtains an infrared divergent solution for the quark-gluon vertex such that Dirac vector and “scalar” components of this vertex are infrared divergent with exponent $-\kappa - \frac{1}{2}$ if either all momenta or the gluon momentum vanish [3]. A numerical solution of a truncated set of Dyson-Schwinger equations confirms this infrared behaviour. The essential components to obtain this solution are the scalar Dirac amplitudes of the quark-gluon vertex and the scalar part of the quark propagator. Both are only present when chiral symmetry is broken, either explicitly or dynamically.

To investigate how this self-consistent quark propagator and quark-gluon vertex solution relates to quark confinement the anomalous infrared exponent of the four-quark function is determined. The static quark potential can be obtained from this four-quark one-particle irreducible Green function, which behaves like $(p^2)^{-2}$ for $p^2 \rightarrow 0$ due to the infrared enhancement of the quark-gluon vertex for vanishing gluon momentum. Using the well-known relation for a function $F \propto (p^2)^{-2}$ one gets

$$V(\mathbf{r}) = \int \frac{d^3 p}{(2\pi)^3} F(p^0 = 0, \mathbf{p}) e^{i\mathbf{p}\mathbf{r}} \sim |\mathbf{r}| \quad (2)$$

for the static quark-antiquark potential $V(\mathbf{r})$. Therefore the infrared divergence of the quark-gluon vertex, as found in the scaling solution of the coupled system of Dyson-Schwinger equations, the vertex overcompensates the infrared suppression of the gluon propagator such that one obtains a linearly rising potential.

DEPENDENCE ON LORENTZ TRANSFORMATION PROPERTIES

The above described mechanism which directly links chiral symmetry breaking with quark confinement raises the question about the role of the Dirac structure in quark confinement. As one expects that fundamental charges are confined by a linear potential a next logical step is to investigate the infrared behaviour of the propagator and the vertex of a fundamentally charged scalar [21, 22].

In contrast to quark Green’s functions the tensor structure of the scalar ones is strongly simplified. Compared to two components in the fermionic propagator, the scalar propagator features only a single structure. Similarly the vertex depending on two independent momenta can be decomposed into two tensors (instead of twelve).

However, a scalar theory has renormalizable self-interactions and therefore the number of terms in the Dyson-Schwinger and Functional Renormalization Group equations are significantly increased. (NB: For the derivation of the Dyson-Schwinger equations one may employ the MATHEMATICA package DoDSE [23]. A package for Functional Renormalization Group equations will be published [24].) First, the uniform scaling limit is studied. Applying the additional constraints on the infrared exponents that arise from the comparison of the inequivalent towers of Dyson-Schwinger and Functional Renormalization Group equations [7, 20], the system of equations for the anomalous exponents simplifies. One finds the scaling and the decoupling solutions with an unaltered Yang-Mills sector. In the case of the scaling solution for a massive scalar the scalar-gluon vertex can show either of two behaviors [21, 22]. In one case, which will be discussed here further, it exhibits the same infrared exponent as the quark-gluon vertex.

However, the uniform scaling displays only part of the potential infrared enhancements. Vertex functions may also become divergent when only a subset of the external momenta vanish. Such kinematic divergences provide a mechanism for the long-range interaction of quarks as described in the section above. To this end it is gratifying to realize that the kinematic divergences of the scalar-gluon vertex are identical to those of the quark-gluon vertex. These singularities induce a confining interaction in the four-scalar vertex function as they did in the case of the four-quark vertex function in the case of scalar QCD. Corresponding infrared leading diagrams are shown in Fig. 1. Their Fourier transform according to eq. (1) leads to a linearly rising static potential.

This result provides the possibility that within functional approaches static confinement is an universal property of the gauge sector even though it is formally represented in the functional equations of the matter sector.

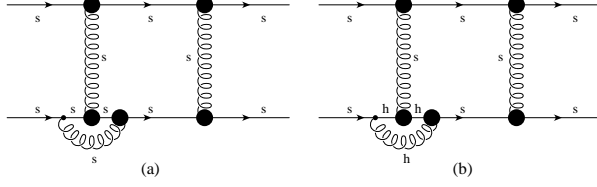


FIGURE 1. Infrared leading diagrams for the four-scalar vertex function in the uniform scaling limit (a) and displaying a kinematic divergence (b).

DEPENDENCE ON THE REPRESENTATION / GROUP

As adjoint charges will not be subject to a linearly rising potential, and as the deviations from the scaling laws are only possible in case of cancellations we have investigated whether such cancellations for different representations and groups occur [25].

Hereby the main difference of course is the color prefactor in the vertex functions: A matter-gluon-vertex of a fundamentally charged field is proportional to a Gell-Mann matrix whereas for an adjoint charge the structure constant appears. Correspondingly, the color algebra changes. *E.g.* the so-called sword-fish diagram is proportional to the Gell-Mann matrix for fundamental scalars but vanishes (due to the antisymmetry of the structure constants) for an adjoint scalar. Thus, for some would-be infrared leading diagrams there are vanishing color prefactors in the adjoint representation.

Noting that the exceptional Lie group G_2 has a trivial center one expects for this group the absence of an (asymptotically) linearly rising potential already for the fundamental representation. And as a matter of fact, we find some systematic cancellations and thus vanishing diagrams for the fundamental representation of G_2 . In the comparison of $SU(N)$ and G_2 this occurs for infrared leading and non-leading diagrams.

In addition, there are significant differences when comparing the unquenched to the quenched scalar-gluon vertex Dyson-Schwinger equation.

SUMMARY

Based on the scaling solution of Functional Equations we have pointed out the possibility that confinement of fundamental charges is related to an infrared divergent matter-gluon vertex. This in turn leads then to a $1/k^4$ singularity in the four-point function and thus to a linearly rising static potential. In addition, for the adjoint representation and for the gauge group G_2 there are systematic cancellations. Whether these will influence the conclusion on the static potential still has to be investigated.

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